Cascade atom in high-Q cavity: the spectrum for non-Markovian decay

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Abstract: Spontaneous emission spectra for an excited three level cascade atom in an empty damped high-Q cavity are determined from the atom-cavity mode zero temperature master equation and the quantum regression theorem. Interference effects are shown.

In a recent paper [1], we considered the non-Markovian decay of a three level cascade atom with both transitions coupled to a single structured reservoir of quantized field modes for both a high-Q cavity and in a photonic band gap system. Based on the approach given in [2], the dynamics of this system was treated via the essential states approach, using Laplace transform methods applied to the coupled amplitude equations. Non-Markovian behaviour for the population dynamics of the atomic system was found, such as oscillatory decay for the high-Q cavity case and population trapping for the photonic band-gap case. A Markovian master equation approach was also applied, in which the atomic system was augmented by a small number of discrete quasimodes or pseudomodes, which in the quasimode treatment themselves undergo Markovian relaxation into a flat reservoir of continuum quasimodes. For the high-Q cavity case a single discrete quasimode was involved, for the PBG case two coupled discrete quasimodes were needed. The essential states and Markoff methods gave identical results, showing that complicated non-Markovian behaviour can be treated by enlarging the non-Markovian system, thereby turning a non-Markovian problem into a Markovian one.

In the present paper we have examined the spontaneous emission spectrum from a cascade atom located inside a high-Q cavity. The cascade atom was initially in its uppermost level and the cavity mode empty of photons, and thus a system with two basic excitations was studied. The cavity mode decays into a zero temperature reservoir of external field modes. The spectrum was examined for three different physical situations regarding the location of an idealised detector atom. In case A the detector atom was located inside the cavity and directly influenced by the cavity field. In case B the detector atom was exposed to the field emerging from a partially transmitting cavity mirror. In case C the detector atom responded directly to sideways emission from the cascade atom inside the cavity. The spectrum was found from the weakly coupled detector atom response and involved in each case Laplace transforms of the density matrix and matrix elements of the evolution operator in a super-operator form. The spectral line-shapes only reflect cavity decay, since the weakly coupled detector atom spectrometer had zero bandwidth. Spectra have been determined for intermediate and strong coupling regime situations, where the two atom-cavity mode coupling constants g_1, g_2 (lower, upper transition) are comparable or large compared to the cavity decay rate Γ. The resonance case, where both atomic transition frequencies ω_1, ω_2 were the same and equal to the cavity frequency ω_c was studied along with cases of non-zero cavity detuning δ ($= \omega_c - \omega_0$) from the average of the atomic transition frequencies ω_0, and cases where the difference $2\delta'$ ($= \omega_1 - \omega_2$) between the two atomic transition frequencies is non-zero.

The spectra for Case A and Case B were found to be essentially the same. The spectral features for Cases B(A) and C were qualitatively similar, with six spectral peaks for resonance cases and eight for detuned cases. These general features of the spectra can be explained via the dressed atom model. In the resonance situation with $\delta=\delta'=0$, spectra in both Case B(A) and Case C are shown below in figures 1 and 2, for several different cavity decay rates traversing the regime from intermediate coupling to strong coupling. Case B(A) and Case C spectra differed in detail. In particular, the spectra for Case C exhibited a deep spectral hole when the spectral frequency was equal to the cavity mode frequency, a feature that persisted, for strong coupling, over a wide range of detuning conditions. This spectral hole was absent for Case A. For intermediate coupling conditions, the spectral hole is suppressed, and in fact the spectrum in Case C is very similar to that of Cases A(B), as seen in figures for $\Gamma=1.0$. In the low-Q limit the cavity field operators can be adiabatically eliminated in favour of the atomic operators, i.e. the cavity field `follows' the atomic state. For this reason we might expect that spectra based on field operators (Case A) and spectra based on the atomic operators (Case C) would become similar in this limit. The spectral hole in Case C for strong coupling was particularly prominent for resonance conditions. In this resonance situation, photons associated with the middle to bottom dressed atom transitions coincide in frequency with photons associated with two of the upper to middle dressed atom transitions, and therefore the origin of a photon detected in the spectrometer cannot be identified. This suggests that an interference effect is occurring, though why it is absent in Case B(A) spectra under strong coupling conditions is still unclear. One clue is that the spectral hole in Case C spectra disappeared when the atomic transition frequencies were quite different and the cavity frequency was resonant with the lower atomic transition frequency. This disappearance was sensitive to both the detunings having opposite sign and a magnitude close to $g=g_1=g_2$. The explanation is expected to be found from an examination of the dressed states in the strong coupling limit.

References

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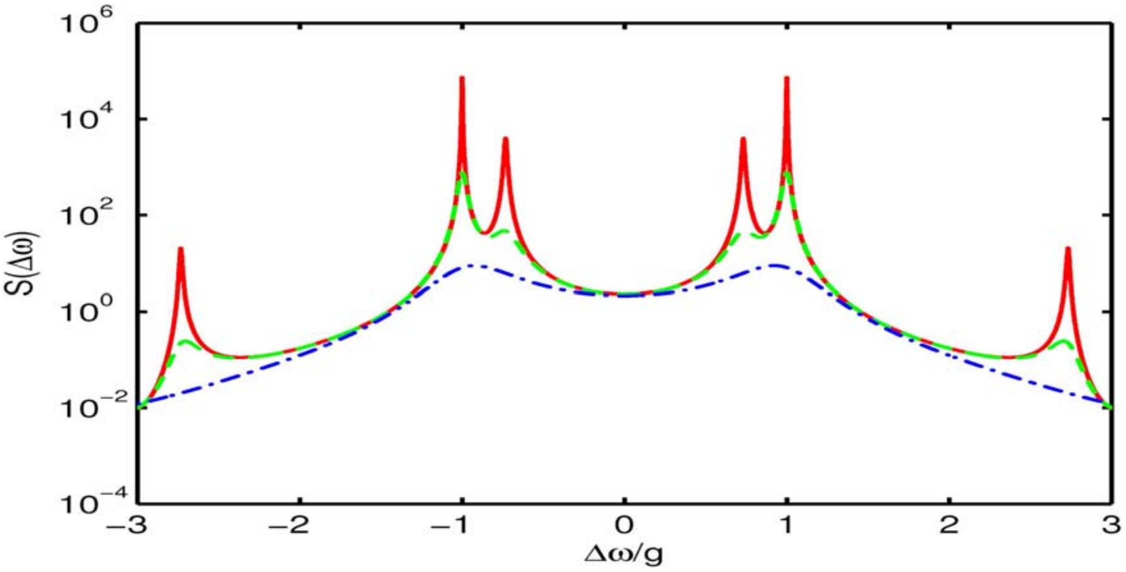

Fig. 1. Cascade atom in high-Q cavity. Spontaneous emission spectra S(ω) versus spectral detuning from cavity frequency $\Delta\omega=(\omega-\omega_c)$. The case of resonance is shown, where each transition is resonant with the cavity frequency, $\delta=\delta'=0$. The cascade atom-cavity mode coupling constants are $g_1=g_2=g=1$ and the detector atom-cavity mode coupling constant is $\mu=1$. The cavity decay $\Gamma=0.01$ is the solid line, $\Gamma=0.1$ is the dashed line and $\Gamma=1.0$ is the chained line. The spectrum for Case A(B) - end emission is shown.

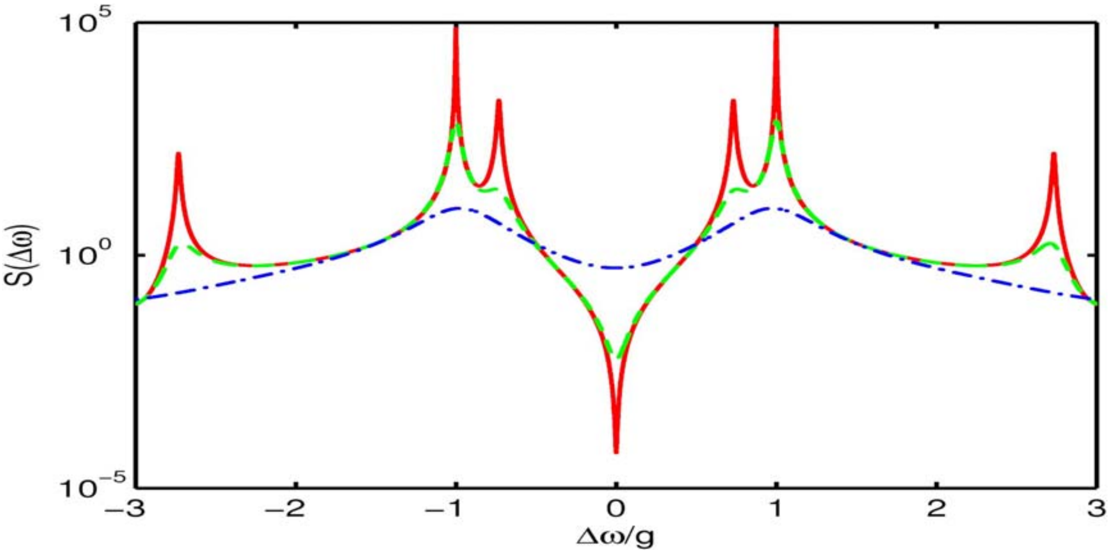

Fig. 2. Cascade atom in high-Q cavity. Spontaneous emission spectra S(ω) versus spectral detuning from cavity frequency $\Delta\omega=(\ \omega-\omega_c\ )$. The case of resonance is shown, where each transition is resonant with the cavity frequency, $\delta=\delta'=0$. The coupling constants are $g_1=g_2=g=1$ and the detector atom coupling constants for the two cascade atom transitions are $R_1=R_2=1$. The cavity decay $\Gamma=0.01$ is the solid line, $\Gamma=0.1$ is the dashed line and $\Gamma=1.0$ is the chained line. The spectrum for Case C - side emission is shown.